COST, REVENUE AND PROFIT FUNCTIONS

Cost functions

Cost is the total cost of producing output.

The cost function consists of two different types of cost:
- Variable costs
- Fixed costs.

Variable cost varies with output (the number of units produced). The total variable cost can be expressed as the product of variable cost per unt and number of units produced. If more items are produced cost is more.

Fixed costs normally donot vary with output. In general these costs must be incurred whether the items are produced or not.

Cost Function

$$C(x) = F + Vx$$

C	= Total cost
F	= Fixed cost
V	= Variable cost Per unit
x	= No of units produced and sold

It is called a **linear cost function.**

Revenue Function

Revenue is the total payment received from selling a good or performing a service.
The revenue function, R(x), reflects the revenue from selling "x" amount of output items at a price of "p" per item.

$$R(x) = px$$

Profit Functions

The Profit function P(x) is the difference between the revenue function R(x) and the total cost function C(x)
Thus P(x) = R(x) − C(x)

$$Profit = Revenue - Cost$$

Profit = Revenue − Cost
$P = R − C$

Examples

1. Assume that fixed costs is Rs. 850, variable cost per item is Rs. 45, and selling price per unit is Rs. 65. Write,

   i. Cost function

   ii. Revenue function

   iii. Profit function

i.	Cost Function	= Variable cost + Fixed cost
		= $45x + 850$
ii.	Revenue function	= $65x$
iii.	Profit function	= $R(x) - TC(x)$
		= $65x - (45x+850)$
		= $20x - 850$

2. Revenue and cost functions for a company are given below.

   Revenue $R(x)$ = $-36x^2 + 2000x$

   Cost $C(x)$ = $125x + 6500$

   Write the simplified form of the profit function

$P(x)$	= $R(x) - C(x) = -36x^2 + 2000x - (125x + 6500)$
	= $-36x^2 + 2000x - 125x - 6500$
	= $-36x^2 + 1875x - 6500$

3. If a retail store has fixed cost of Rs.150 and variable cost per unit is Rs.175 and sells its product at Rs. 500 per unit.

   i. Find the cost function C(x)

   ii. What would the revenue function be?

   iii. What would the profit function be?

i.   Cost function $= C(x) = 175\,x + 150$

ii.   Revenue Function $= R(x) = 500\,x$

iii.   Profit function   =   Revenue function – Cost function

                               = $R(x) - C(x)$

                               = $(500\,x) - (175\,x + 150)$

                               = $500\,x - 175\ -\ 150$

                               = $325\,x - 150$

4.   A company produces and sells a product and fixed costs of the company are Rs. 6,000 and variable cost is Rs. 25 per unit, and sells the product at Rs. 50 per unit.

    i   Find the total cost function.

    Ii   Find the total revenue function.

    iii   Find the profit function, and determine the profit when 1000 units are sold.

    iv   How many units have to be produced and sold to yield a profit of Rs. 10,000?

i. $TC = 6000 + 25\,x$

ii. $TR = 50\,x$

iii. $P = 50\,x - (6000 + 25\,x) = 25\,x - 6000$

P $= 25 \times 1000 - 6000 = 19{,}000$

iv.   P $= 10{,}000$

        $10{,}000 = 25\,x - 6{,}000$

                $x = 16000/25$

                $x = 640$ units

5.   A firms demand function for a good is given by $D = 100 - 2\,x$ and their total cost function is given by $TC = 300 + 8\,x$.

        Find the profit Function

        Total Revenue $= D.\,x$

        TR $= (100 - 2\,x)*\,x = 100\,x - 2\,x^2$

        TC $= 300 + 8\,x$

Profit = TR-TC

Profit $= 100x - 2x^2 - 300 - 8x = -2x^2 + 92x - 300$

Marginal Cost (MC)

Additional cost of producing an additional unit of Output is called ***marginal cost.***

Marginal cost is the change in total cost that arises when the quantity produced changes by one unit.

Marginal cost (MC) function is expressed as the first derivative of the total cost (TC) function with respect to quantity (x).

Marginal Revenue (MR)

Marginal revenue is the change in total revenue from increasing quantity by one unit.

Marginal revenue (MR) is the additional revenue that will be generated by increasing product sales by one unit.

Marginal Revenue (MR) function is expressed as the first derivative of the Revenue function (TR) with respect to quantity (x).

Examples

1. Cost function of a company is $6x^2 + 5x + 100$. Calculate the marginal cost function.

   Answer

   $C(x) = 6x^2 + 5x + 100$

   $\underline{dc} = MC = 2*6x + 5 + 0$

   dx

   MC   = $12x + 5$

The marginal cost at $x = 10$

   MC   = $12*10 + 5 = 125$

2. The total cost function for manufacturing x shoes per year is given by

   $C(x) = 525 + 150x - 0.2x^2$

   Calculate the marginal cost

Answers

i.   The marginal cost is

   $C(x) = 525 + 150x - 0.2x^2$

   $dc/dx = MC = 150 - 0.4x$

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3. The market department of ABC company recommends manufacture and market a new school bag. The financial department provides the following cost function (in Rs)

$C(x)$ = 600 + 120 x where Rs 600 is the estimated fixed costs and the estimated variable cost is Rs. 120 per bag. The revenue function (in Rs.) is $R(x) = 15\,x + 0.005\,x^2$

  i. Find the marginal cost (MC) function

ii. Find the marginal revenue (MR) function

Answers

i.

    $C(x)$ = 600 + 120 x

    dc/d x =   MC   =   120

ii. $R(x)$ = $15\,x + 0.005\,x^2$

    dr/d x =   MR   =   15+ (0.05 x 2) x

                             15 +0.01 x

4. The demand function of company is p = 42 -0.001 x and cost function is $C(x)$ = 30x +1200, where x is the number of units demanded.

i.   Find the profit function

ii.   Find the marginal profit Function

iii   Calculate the profit for 1000 units

Answers

i.   First you have to find the revenue function

    $R(x)$ = p* x

    = (42 -0.001 x)* x

    = $42\,x - 0.001\,x^2$

Profit function =   Revenue function – Cost Function

          = $42\,x - 0.001\,x^2$- (30 x + 1,200)

                  $42\,x$ -0.001 x^2- 30 x - 1,200

          =   -0.001 x^2 + 12 x -1200

ii.   Find the Marginal Profit function

To find the marginal profit, we need to take the derivative of the profit function.

$$\frac{dp}{dx} = MP = -0.002x + 12$$

iii $P = -0.001x^2 + 12x - 1200$

$= -0.001(1000)^2 + 12 \times 1000 - 1200$

$= 9800$

PROFIT MAXIMIZATION
Profit maximization rule:

MR=MC or Differentiation of Profit Function

Examples

1. If, Demand Function is D = 160 - 0.0025x and Cost function is C = 15x + 0.0025x^2

Find the profit revenue-maximizing output level

Answer

$R = D * x$

Revenue Function   = $(160 - 0.0025x) * x$

= $160x - 0.0025x^2$

Cost Function   = $15x + 0.0025x^2$

At the level of profit maximizing, MC = MR

MR = MC

MR   = $160 - 2*0.0025x$

= $160 - 0.005x$

MC   = $15 + 2*0.0025x$

= $15 + 0.005x$

MR = MC

$160 - 0.005\,x = 15 + 0.005\,x$

$160 - 15 = 0.005\,x + 0.005\,x$

$0.01\,x = 145$

$x = 145/0.01$

$x = 14{,}500$ Units

2. Weekly profit Function of a company is given by $P = 1400q - q^2 - 240{,}000$ where q is the number of units produced per week. Calculate the number of units to be sold to maximizing the weekly profit.

Answer
Differentiation of profit Function

$P = 1400q - q^2 - 240{,}000$

$dp/dq = 1400 - 2q - 0$
$= 1400 - 2q$
$2q = 1400$
$q = 700$ Units